# EE3124 Tutorial 1 (Solution)

# Basic Magnetics

**Name:** **Student No.:**

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**Q1** - A motor's shaft is spinning at a speed of 1800 r/min. What is the shaft speed in radians per second?

**Solution**

$$\omega = (1800 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right) = 188.5 \text{ rad/s}$$

**Q2** - A motor is supplying 50 Nm of torque to its load. If the motor's shaft is turning at 1500 r/min, what is the mechanical power supplied to the load in watts? In horsepower?

**Solution**

The mechanical power supplied to the load is

$$P = \tau\omega = (50 \text{ N}\cdot\text{m})(1500 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right) = 7854 \text{ W}$$

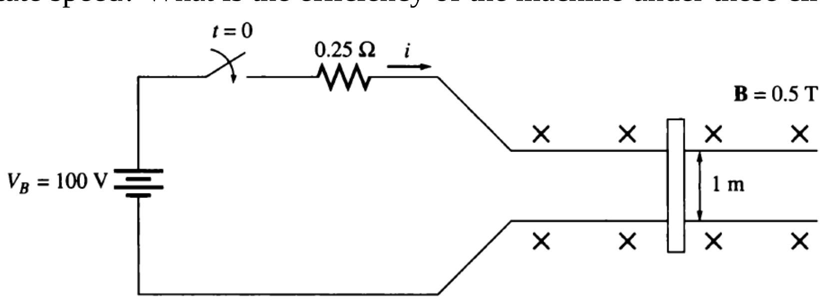
$$P = (7854 \text{ W})\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = 10.5 \text{ hp}$$

**Q3** - A linear machine shown as follows has a magnetic flux density of 0.5 T directed into the page, a resistance of 0.25 Ω, a bar length l = 1.0 m, and a battery voltage of 100 V.

(a) What is the initial force on the bar at starting? What is the initial current flow?

(b) What is the no-load steady-state speed of the bar?

(c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady-state speed? What is the efficiency of the machine under these circumstances?



**Solution**

*(a)* The current in the bar at starting is

$$i = \frac{V_B}{R} = \frac{100\ \text{V}}{0.25\ \Omega} = 400\ \text{A}$$

Therefore, the force on the bar at starting is

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) = (400\ \text{A})(1\ \text{m})(0.5\ \text{T}) = 200\ \text{N, to the right}$$

*(b)* The no-load steady-state speed of this bar can be found from the equation

$$V_B = e_{\text{ind}} = vBl$$

$$v = \frac{V_B}{Bl} = \frac{100\ \text{V}}{(0.5\ \text{T})(1\ \text{m})} = 200\ \text{m/s}$$

*(c)* With a load of 25 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_{\text{app}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{app}}}{Bl} = \frac{25\ \text{N}}{(0.5\ \text{T})(1\ \text{m})} = 50\ \text{A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100\ \text{V} - (50\ \text{A})(0.25\ \Omega) = 87.5\ \text{V}$$

and the velocity of the bar will be

$$v = \frac{V_B}{Bl} = \frac{87.5\ \text{V}}{(0.5\ \text{T})(1\ \text{m})} = 175\ \text{m/s}$$

The *input* power to the linear machine under these conditions is

$$P_{\text{in}} = V_B i = (100\ \text{V})(50\ \text{A}) = 5000\ \text{W}$$

The *output* power from the linear machine under these conditions is

$$P_{\text{out}} = V_B i = (87.5\ \text{V})(50\ \text{A}) = 4375\ \text{W}$$

Therefore, the efficiency of the machine under these conditions is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{4375\ \text{W}}{5000\ \text{W}} \times 100\% = 87.5\%$$

**Q4 -** A linear machine (same topology as the upper figure) has the following characteristics:

| | |
|---|---|
| B = 0.5 T into page | R = 0.25 Ω |
| l = 0.5 m | VB= 120 V |

(a) If this bar has a load of 20 N attached to it opposite to the direction of motion, what is the steady-state speed of the bar?

(b) If the bar runs off into a region where the flux density falls to 0.45 T, what happens to the bar? What is its final steady-state speed?

(c) Suppose VB is now decreased to 100 V with everything else remaining as in part (b). What is the new steady-state speed of the bar?

(d) From the results for parts (b) and (c), what are two methods of controlling the speed of a linear machine (or a real dc motor)?

**Solution**

*(a)* With a load of 20 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_{\text{app}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{app}}}{Bl} = \frac{20\text{ N}}{(0.5\text{ T})(0.5\text{ m})} = 80\text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 120\text{ V} - (80\text{ A})(0.25\ \Omega) = 100\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{100\text{ V}}{(0.5\text{ T})(0.5\text{ m})} = 400\text{ m/s}$$

*(b)* If the flux density drops to 0.45 T while the load on the bar remains the same, there will be a speed transient until $F_{\text{app}} = F_{\text{ind}} = 20\text{ N}$ again. The new steady state current will be

$$F_{\text{app}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{app}}}{Bl} = \frac{20\text{ N}}{(0.45\text{ T})(0.5\text{ m})} = 88.9\text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 120\text{ V} - (88.9\text{ A})(0.25\ \Omega) = 97.8\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{97.8\text{ V}}{(0.45\text{ T})(0.5\text{ m})} = 433\text{ m/s}$$

*(c)* If the battery voltage is decreased to 100 V while the load on the bar remains the same, there will be a speed transient until $F_{\text{app}} = F_{\text{ind}} = 20\text{ N}$ again. The new steady state current will be

$$F_{\text{app}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{app}}}{Bl} = \frac{20\text{ N}}{(0.45\text{ T})(0.5\text{ m})} = 88.9\text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100\text{ V} - (88.9\text{ A})(0.25\ \Omega) = 77.8\text{ V}$$

and the velocity of the bar will be

$$v = \frac{e_{\text{ind}}}{Bl} = \frac{77.8\text{ V}}{(0.45\text{ T})(0.5\text{ m})} = 344\text{ m/s}$$

*(d)* From the results of the two previous parts, we can see that there are two ways to control the speed of a linear dc machine. *Reducing* the flux density $B$ of the machine *increases* the steady-state speed, and *reducing* the battery voltage $V_B$ *decreases* the stead-state speed of the machine. Both of these speed control methods work for real dc machines as well as for linear machines.

**Q5** - A core with three legs is shown as follows. Its depth is 5 cm, and there are 400 turns on the center leg. The remaining dimensions are shown in the figure. The core is composed of a steel having the magnetization curve as follows. Answer the following questions about this core:

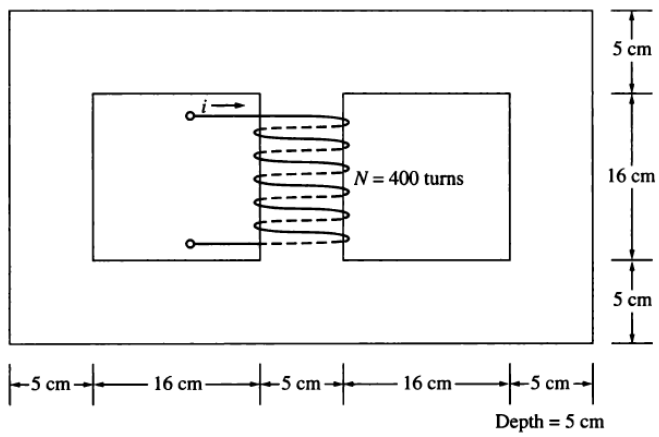
(a) What current is required to produce a flux density of 0.5 T in the central leg of the core?

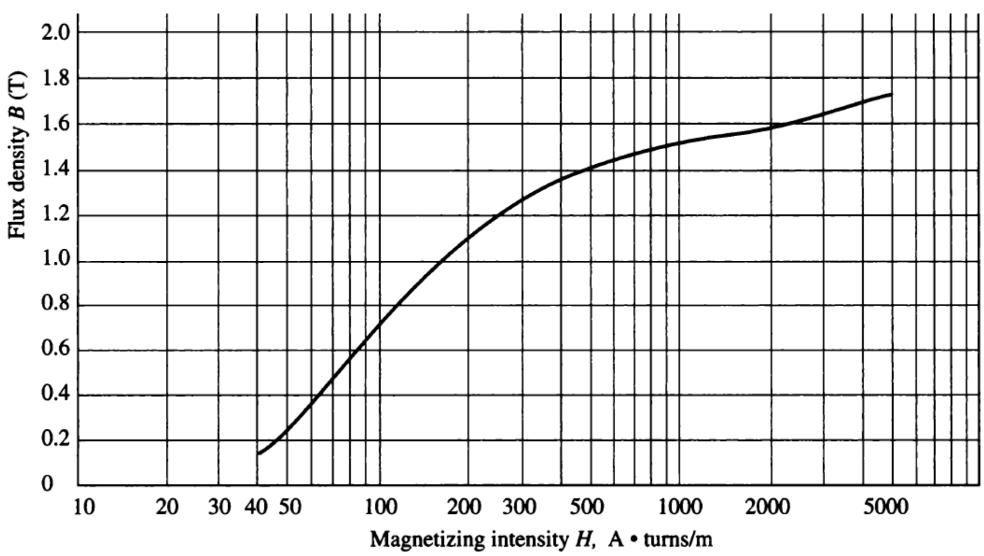
(b) What current is required to produce a flux density of 1.0 T in the central leg of the core? Is it twice the current in part (a)?

(c) What are the reluctances of the central and right legs of the core under the conditions in part (a)?

(d) What are the reluctances of the central and right legs of the core under the conditions in part (b)?

(e) What conclusion can you make about reluctances in real magnetic cores?



## Solution

*(a)* A flux density of 0.5 T in the central core corresponds to a total flux of

$$\phi_{\text{TOT}} = BA = (0.5\text{ T})(0.05\text{ m})(0.05\text{ m}) = 0.00125\text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.000625\text{ Wb}$, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.000625\text{ Wb}}{(0.05\text{ m})(0.05\text{ m})} = 0.25\text{ T}$$

The magnetizing intensity $H$ required to produce a flux density of 0.25 T can be found from Figure
It is 50 A·t/m. Similarly, the magnetizing intensity $H$ required to produce a flux density of 0.50 T is 75 A·t/m. The mean length of the center leg is 21 cm and the mean length of each outer leg is 63 dm, so the total MMF needed is

$$\mathcal{F}_{\text{TOT}} = H_{\text{center}}\, l_{\text{center}} + H_{\text{outer}}\, l_{\text{outer}}$$
$$\mathcal{F}_{\text{TOT}} = (75\text{ A}\cdot\text{t/m})(0.21\text{ m}) + (50\text{ A}\cdot\text{t/m})(0.63\text{ m}) = 47.3\text{ A}\cdot\text{t}$$

and the required current is

$$i = \frac{\mathcal{F}_{\text{TOT}}}{N} = \frac{47.3\text{ A}\cdot\text{t}}{400\text{ t}} = 0.12\text{ A}$$

*(b)* A flux density of 1.0 T in the central core corresponds to a total flux of

$$\phi_{\text{TOT}} = BA = (1.0\text{ T})(0.05\text{ m})(0.05\text{ m}) = 0.0025\text{ Wb}$$

By symmetry, the flux in each of the two outer legs must be $\phi_1 = \phi_2 = 0.00125\text{ Wb}$, and the flux density in the other legs must be

$$B_1 = B_2 = \frac{0.00125\text{ Wb}}{(0.05\text{ m})(0.05\text{ m})} = 0.50\text{ T}$$

The magnetizing intensity $H$ required to produce a flux density of 0.50 T can be found from Figure
It is 75 A·t/m. Similarly, the magnetizing intensity $H$ required to produce a flux density of 1.00 T is about 160 A·t/m. Therefore, the total MMF needed is

$$\mathcal{F}_{\text{TOT}} = H_{\text{center}} I_{\text{center}} + H_{\text{outer}} I_{\text{outer}}$$
$$\mathcal{F}_{\text{TOT}} = (160\text{ A}\cdot\text{t/m})(0.21\text{ m}) + (75\text{ A}\cdot\text{t/m})(0.63\text{ m}) = 80.8\text{ A}\cdot\text{t}$$

and the required current is

$$i = \frac{\phi_{\text{TOT}}}{N} = \frac{80.8\text{ A}\cdot\text{t}}{400\text{ t}} = 0.202\text{ A}$$

This current is *not* twice the current in part *(a)*.

*(c)* The reluctance of the central leg of the core under the conditions of part *(a)* is:

$$\mathcal{R}_{\text{cent}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(75\text{ A}\cdot\text{t/m})(0.21\text{ m})}{0.00125\text{ Wb}} = 12.6\text{ kA}\cdot\text{t/Wb}$$

The reluctance of the right leg of the core under the conditions of part *(a)* is:

$$\mathcal{R}_{\text{right}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(50\text{ A}\cdot\text{t/m})(0.63\text{ m})}{0.000625\text{ Wb}} = 50.4\text{ kA}\cdot\text{t/Wb}$$

*(d)* The reluctance of the central leg of the core under the conditions of part *(b)* is:

$$\mathcal{R}_{\text{cent}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(160\ \text{A}\cdot\text{t/m})(0.21\ \text{m})}{0.0025\ \text{Wb}} = 13.4\ \text{kA}\cdot\text{t/Wb}$$

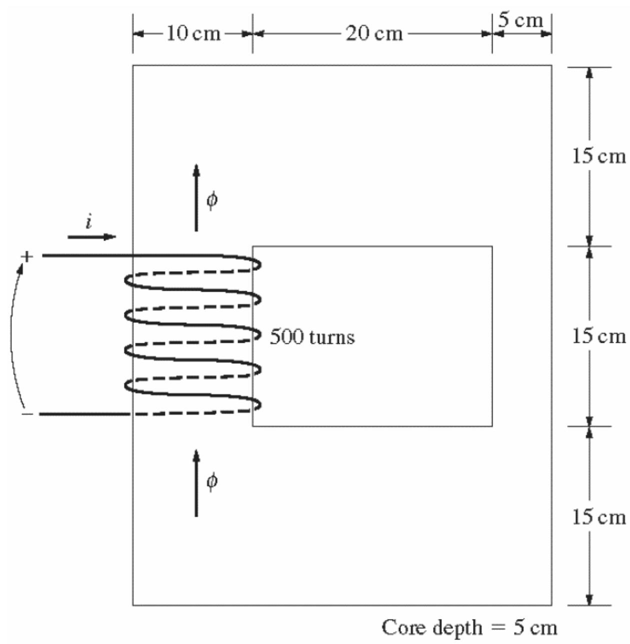
The reluctance of the right leg of the core under the conditions of part *(b)* is:

$$\mathcal{R}_{\text{right}} = \frac{\mathcal{F}_{\text{TOT}}}{\phi_{\text{TOT}}} = \frac{(75\ \text{A}\cdot\text{t/m})(0.63\ \text{m})}{0.00125\ \text{Wb}} = 37.8\ \text{kA}\cdot\text{t/Wb}$$

*(e)* The reluctances in real magnetic cores are not constant.

**Q6** - A ferromagnetic core is shown as follows. The depth of the core is 5 cm. The other dimensions of the core are as shown in the figure. Assume that the relative permeability of the core is 800.

1) Find the value of the current that will produce a flux of 0.005 Wb.
2) With this current, what is the flux density at the top of the core?
3) What is the flux density at the right side of the core?



SOLUTION There are three regions in this core. The top and bottom form one region, the left side forms a second region, and the right side forms a third region. If we assume that the mean path length of the flux is in the center of each leg of the core, and if we ignore spreading at the corners of the core, then the path lengths are $l_1 = 2(27.5 \text{ cm}) = 55 \text{ cm}$, $l_2 = 30 \text{ cm}$, and $l_3 = 30 \text{ cm}$. The reluctances of these regions are:

$$\mathcal{R}_1 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.55 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.15 \text{ m})} = 72.9 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.10 \text{ m})} = 59.7 \text{ kA} \cdot \text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l}{\mu A} = \frac{l}{\mu_r \mu_o A} = \frac{0.30 \text{ m}}{(800)(4\pi \times 10^{-7} \text{ H/m})(0.05 \text{ m})(0.05 \text{ m})} = 119.4 \text{ kA} \cdot \text{t/Wb}$$

The total reluctance is thus

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \mathcal{R}_2 + \mathcal{R}_3 = 72.9 + 59.7 + 119.4 = 252 \text{ kA} \cdot \text{t/Wb}$$

and the magnetomotive force required to produce a flux of 0.005 Wb is

$$\mathcal{F} = \phi\, \mathcal{R} = (0.005 \text{ Wb})(252 \text{ kA} \cdot \text{t/Wb}) = 1260 \text{ A} \cdot \text{t}$$

and the required current is

$$i = \frac{\mathcal{F}}{N} = \frac{1260 \text{ A} \cdot \text{t}}{500 \text{ t}} = 2.5 \text{ A}$$

The flux density on the top of the core is

$$B = \frac{\phi}{A} = \frac{0.005 \text{ Wb}}{(0.15 \text{ m})(0.05 \text{ m})} = 0.67 \text{ T}$$

The flux density on the right side of the core is

$$B = \frac{\phi}{A} = \frac{0.005 \text{ Wb}}{(0.05 \text{ m})(0.05 \text{ m})} = 2.0 \text{ T}$$